**King Fahd University of Petroleum & Minerals**

**College of Computer Sciences & Engineering**

**Department of Information and Computer Science**

**ICS 253: Discrete Structures I**

**Final Exam – 131**

**120 Minutes**

Instructors: Dr. Husni [Section 1] Dr. Abdulaziz [Section 2] Dr. Wasfi [Section 3]

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| **Question** | **Max** | **Earned** |
| 1 | 21 |  |
| 2 | 9 |  |
| 3 | 7 |  |
| 4 | 7 |  |
| 5 | 24 |  |
| 6 | 18 |  |
| 7 | 7 |  |
| 8 | 7 |  |
| **Total** | **100** |  |

**Wednesday, January 1, 2014**

**«Sec»-«S» «Student\_Name» <«Stuid»>**

**Question 1: [21 Points]** Indicate whether the given sentence is true or false. In the answer column write either ✓ for "true" or 🗶 for "false".

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| **Statement** | **Answer** |
| 1. The negation of the proposition “Ahmad’s PC runs Linux” is “Ahmad’s PC runs Windows”.
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| 1. The contrapositive of the conditional statement “The home team wins whenever it is raining?” is “If the home team does not win, then it is not raining.”
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| 1. Logical connectives are used extensively in searches of large collections of information, such as indexes of Web pages.
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| 1. ￢*(p* → *q)* and *p* ∧￢*q* are logically equivalent.
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| 1. If *P(x)* is the statement “*x* + 1 *> x*” where the domain consists of all real numbers, then ∀*x P(x) is false.*
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| 1. The quantifiers ∀ and ∃ have higher precedence than all logical operators from propositional calculus.
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| 1. ∀*x(P(x)* ∧ *Q(x))* and∀*xP(x)* ∧∀*xQ(x)* are logically equivalent.
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| 1. the statement ∀x∃y(x + y = 0) says that every real number has an additive inverse.
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| 1. An *onto* function *f* :*A*→*B* maps the *A* over a piece of the set *B*, not over the *entirety* of it.
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| 1. If *x* and *y* are integers and both *xy* and *x* + *y* are even, then both *x* and *y* are odd.
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| 1. The set of all positive integers less than 100 can be denoted by {1, 4, 5, . . . , 99}.
 |  |
| 1. If A = {1, 2}, then A2 = {(1, 1), (1, 2), (2, 1), (2, 2)}.
 |  |
| 1. If *A1* = {0*,* 2*,* 4*,* 6*,* 8}, *A2* = {0*,* 1*,* 2*,* 3*,* 4}*,* and *A3* = {0*,* 3*,* 6*,* 9}, then $\bigcap\_{i=1}^{3}A\_{i}=\left\{0\right\}.$
 |  |
| 1. The function f (x) = x2 from the set of integers to the set of integers is one-to-one.
 |  |
| 1. A function is not invertible if it is not a one-to-one correspondence.
 |  |
| 1. $\left⌊2x\right⌋=\left⌊x\right⌋+\left⌊x+\frac{1}{2}\right⌋$
 |  |
| 1. The formula for the sequence 1, 1/2, 1/4, 1/8, 1/16, … is *an* = 1*/*4*n*, *n* = 0, 1, 2, …
 |  |
| 1. $\sum\_{j=1}^{5}j^{2}=\sum\_{k=0}^{4}(k+1)^{2} $
 |  |
| 1. If *A* and *B* are countable sets, then *A* ∪ *B* is also countable.
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| 1. Mathematical induction can be used to prove mathematical statements that assert a property is true for all positive integers such as “for every positive integer *n: n! ≤ nn.*”
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| 1. There are 1000 positive integers devisable by 9 between 1000 and 9999 inclusive.
 |  |

**Question 2: [9 Points]** Fill in the first column of the table below by writing the number of the ***most*** **proper** text from the **3rd column** that is related to the text in the **2nd column** (only one number per entry):

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Number** |  | **2nd Column** |  | **3rd Column** |
|  |  | If *k* is a positive integer and *k* +1 or more objects are placed into *k* boxes, then there is at least one box containing two or more of the objects. |  | 1. The Pigeonhole Principle
 |
|  |  | The number of *r*-combinations of a set with *n* distinct elements (*C(n, r) or* $\left(\genfrac{}{}{0pt}{}{n}{r}\right)$) |  | 1. Application of Probability Theory
 |
|  |  | Determining the number of moves in the Tower of Hanoi puzzle and counting bit strings with certain properties. |  | 1. Binomial Coefficients
 |
|  |  | The inductive step shows that if *P(j)* is true for all positive integers not exceeding *k*, then *P(k* +1*)* is true. |  | 1. Applications of Recurrence Relations
 |
|  |  | The inductive step shows that if the inductive hypothesis *P(k)* is true, then *P(k* + 1*)* is also true. |  | 1. Rules of Inference
 |
|  |  | Extensively applied in the study of genetics, where it can be used to help understand the inheritance of traits. |  | 1. Mathematical Induction
 |
|  |  | Basic tools for establishing the truth of statements. They are templates for constructing valid arguments. |  | 1. Strong Induction
 |
|  |  | ∀*y*∃*x*∃*z(T (x, y, z)* ∨ *Q(x, y))* |  | 1. Nested Quantifiers
 |
|  |  | Collection of Objects. |  | 1. Sets
 |

**Question 3: [7 Points] Strong Induction**

Using strong induction, prove that for each positive integer *n*, the *n*th Fibonacci number *fn* is less than (7/4)*n*. Note that Fibonacci numbers are defined as follows: *f*0=1, *f*1=1 and *f*n = *fn*-1 + *fn*-2, *n* ≥ 2.

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**Question 4: [7 Points] Structural Induction**

Let *S* be the subset of the set of ordered pairs of integers defined recursively by

*Basis step:* (0*,* 0)∈ *S*.

*Recursive step:* If (*a, b*)∈ *S*, then (*a, b* + 1)∈ *S*, (*a* + 1*, b* + 1)∈ *S*, and (*a* + 2*, b* + 1)∈ *S*.

Use structural induction to show that *a* ≤ 2*b* whenever (*a, b*) ∈ *S*.

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**Question 5: [24 Points] Counting and Applications**

1. **[5 points]** How many positive integers not exceeding 200 are divisible by 4 or 5?

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1. **[5 points]** What is the coefficient of $x^{13}y^{37}$ in the expansion of $\left(3y-x\right)^{50}$?

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1. **[7 points]** How many bit strings of length 12 have exactly four 1s such that all the 1s are separated by 0s (so no two 1s are adjacent)?

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1. **[7 points]** Recall that a set of 52 playing cards is divided equally into 4 suits. Use the pigeonhole principle to find an expression for the least number of cards required to ensure that at least *x* cards are of the same suit, where $1\leq x\leq 13$.

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**Question 6: [18 Points] Discrete Probability**

1. **[12 points]** A bit string of length six is picked at random such that all bit strings are equally likely. Consider the following events:

*E*1: the bit string begins with 1;

*E*2: the bit string ends with 1;

*E*3: the bit string has exactly three 1s.

* 1. **[4 points]** Find *p*(*E*1 | *E*3).

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* 1. **[2 points]** Are *E*1 and *E*3 independent? Justify.

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* 1. **[6 points]** Are *E*1 and *E*2 independent? Justify.

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1. **[6 points]** An unfair coin is flipped ten times, where *p*(heads) = 3/4. Find the following:

*p*(at least 3 heads appear | at least 1 head appears).

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**Question 7: [7 Points] Applications of Recurrence Relations**

1. **[4 points]** Find a recurrence relation for the number of ternary strings of length *n* that contain two consecutive 0s.

Note: Ternary strings are strings with characters from {0,1,2}.

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1. **[1 point]** What are the initial conditions?

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1. **[2 points]** How many ternary strings of length five contain two consecutive 0s?

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**Question 8: [7 Points] Solving Linear Recurrence Relations**

Solve the following linear recurrence relation, together with the initial conditions given.

$a\_{n}=-6a\_{n-1}-9a\_{n-2}$ for $n\geq 2$ and $a\_{0}=3 and a\_{1}=-3$

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**Some Useful Formulas**

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|  | Addition |  | Modus Tollens |
|  | Simplification |  | Hypothetical syllogism |
|  | Conjunction |  | Disjunctive syllogism |
|  | Modus Ponens |  | Resolution |

